CONDUCTOR LOSS IN HOLLOW WAVEGUIDES USING A SURFACE INTEGRAL FORMULATION

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ABSTRACT

The power-loss method along with a surface integral formulation has been used to compute the attenuation constant in hollow waveguides of arbitrary cross-section. An E-field integral equation has been developed for the surface electric currents which has been transformed into a matrix equation using method of moments. An iterative technique, i.e. Muller's method has been used to obtain the relation between the propagation constant and frequency. The attenuation constants have been calculated and formulated for various waveguides and are in good agreement with published data.

Introduction

Numerous papers are available in the literature for the analysis of waves propagating in hollow waveguides of arbitrary cross-section [1-4]. Some of the papers in this area are the works done by Swaminathan et al [1], Spielman and Harrington [2], Bristol [3] and Kim et al [4]. These papers however deal with hollow waveguides made up of perfectly conducting walls supporting waves at low frequencies. The work presented here is an extension of [1] and deals with the computation of the attenuation constant of hollow waveguides supporting waves at high frequencies.

At millimeter wave frequencies, the finite conductivity of the waveguide walls in hollow waveguides produces an attenuation in the wave propagating in the waveguide. To accurately characterize the hollow waveguide at millimeter wave frequencies, an estimate for the attenuation constant is necessary. Since the finite conductivity of the waveguide walls produces this attenuation, the conductivity of the waveguide walls has to be taken into consideration while calculating the fields produced by the wave propagating in the waveguide. As long as this conductor loss is small, the power-loss method can be used to compute the attenuation constant [5]. The power-loss method coupled with the surface integral formulation [1,7] has been used in this paper to analyze hollow waveguides of arbitrary cross-section.

Formulation

The power-loss method which has been used in this paper for calculating the attenuation constant assumes that the losses are low at high frequencies. Hence it can be safely assumed that the finite conductivity of the waveguide walls has only a small effect on the field configuration within the waveguide. Due to the large conductivity of the waveguide walls, the magnetic field tangential to the wall depends only slightly on the wall conductivity. Thus the tangential magnetic field strength computed for perfectly conducting walls remains the same as the tangential magnetic field strength computed for waveguides made up of walls with finite conductivity.

Based on the power-loss method [5], the attenuation constant is defined as

$$\alpha = \frac{P_L}{2P_T} \tag{1}$$

where

$$P_L = \frac{1}{2} R_s \oint_c |H_{tan}|^2 dl$$

$$P_T = \frac{1}{2} \iint_s \mathrm{Re}(\bar{E} \times \bar{H}^*).\bar{z}ds$$

In the above equations, P_L is the power lost per unit length, P_T is the power transmitted, H_{tan} is the magnetic field tangential to the waveguide walls assuming that the walls are perfectly conducting, R_s is the surface resistance of the guide wall, $\bar{E}$ is the electric field inside the waveguide and $\bar{H}^*$ is the complex conjugate of the magnetic field existing inside the waveguide. As is obvious from the above equations, P_L is given by a contour integral and P_T by a surface integral. The surface resistance R_s at any angular frequency ω is given by

$$R_s = \frac{\omega\mu_o}{2\sigma} \tag{2}$$

where μ_o is the free space permeability and σ is the conductivity of the waveguide walls. Equation (1) represents the formula for computing the attenuation constant of a hollow waveguide with finite cross-section and infinite along the direction of propagation.

To calculate the fields existing in the waveguide made up of walls with finite conductivity a surface integral technique has been utilized [1]. The hollow conducting waveguides are assumed to be infinite in the z-direction and to have arbitrary cross section. The waveguide is completely filled with homogeneous dielectric (air in this case).

Using the surface equivalence principle the waveguide walls have been replaced by equivalent surface electric currents radiating into free-space. Enforcing the appropriate boundary condition, i.e., the total tangential electric field vanishes on the surface of the waveguide the following E-field integral equation has been developed:

$$\bar{n} \times \bar{E}(\bar{J}) = 0 \qquad \text{on } C \qquad (3)$$

where $\bar{E}$ is the scattered field produced by the electric current $\bar{J}$ and $\bar{n}$ is the unit vector normal to the surface of the waveguide. Since TM_z and TE_z modes can propagate in the waveguide, $\bar{E}$ represents the axial electric field for TM_z modes and represents the transverse electric field for TE_z modes.

Method of moments ([8]) with pulse expansion and point matching testing procedure has been used to transform the integral equation into a matrix one. At a fixed wavenumber (β) and angular frequency (ω) the integral equation reduces to the form

$$[Z][I] = [0] \qquad (4)$$

where [Z] is the impedance matrix and [I] is a vector containing the expansion coefficients.

The next step in the calculation of α is to obtain a relation between β and ω using equation (4). For a non-trivial solution to exist for the vector [I], the matrix [Z] has to be singular [1]. The matrix equation can be rewritten in a simpler form for computational purposes as

$$[Z][I] = \lambda_{min}[I] \qquad (5)$$

where λ_{min} is the minimum eigenvalue of the matrix [Z] and [I] is the corresponding eigenvector. Assuming β is fixed, then the frequency ω at which λ_{min} is the smallest gives a relation between β and ω. The Muller's method has been used to find ω at which λ_{min} goes to zero [1,7].

Once the β-ω relation is known, the fields inside and on the surface of the waveguide can be calculated using the eigenvector [I].

Results

The results obtained using the power-loss method along with a surface integral formulation have been compared with analytical results given in [5] for the waveguide shown in Fig.1. The attenuation constant computed for the first TM_z and first TE_z mode propagating in the waveguide are shown in Fig. 2. The cutoff wavelength λc for TM_z mode is 1.789cm and for the TE_z mode is 4.0cm. The results obtained compare very well with the analytical results.

An L Shaped Waveguide is shown in Fig. 3. The attenuation for the first TM_z mode and the first TE_z mode are shown in Fig. 4. The cutoff wavelength λc for the TM_z mode is 1.286cm and for the TE_z mode is 3.321cm.

The surface integral technique along with the power-loss method is a very powerful method for computing the attenuation constant of waveguides with arbitrary cross-sections.

References

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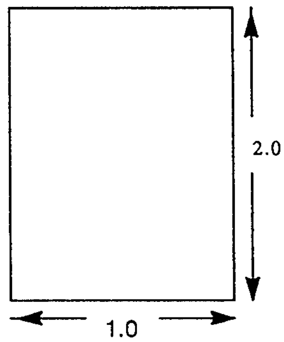

Fig. 1: Rectangular waveguide

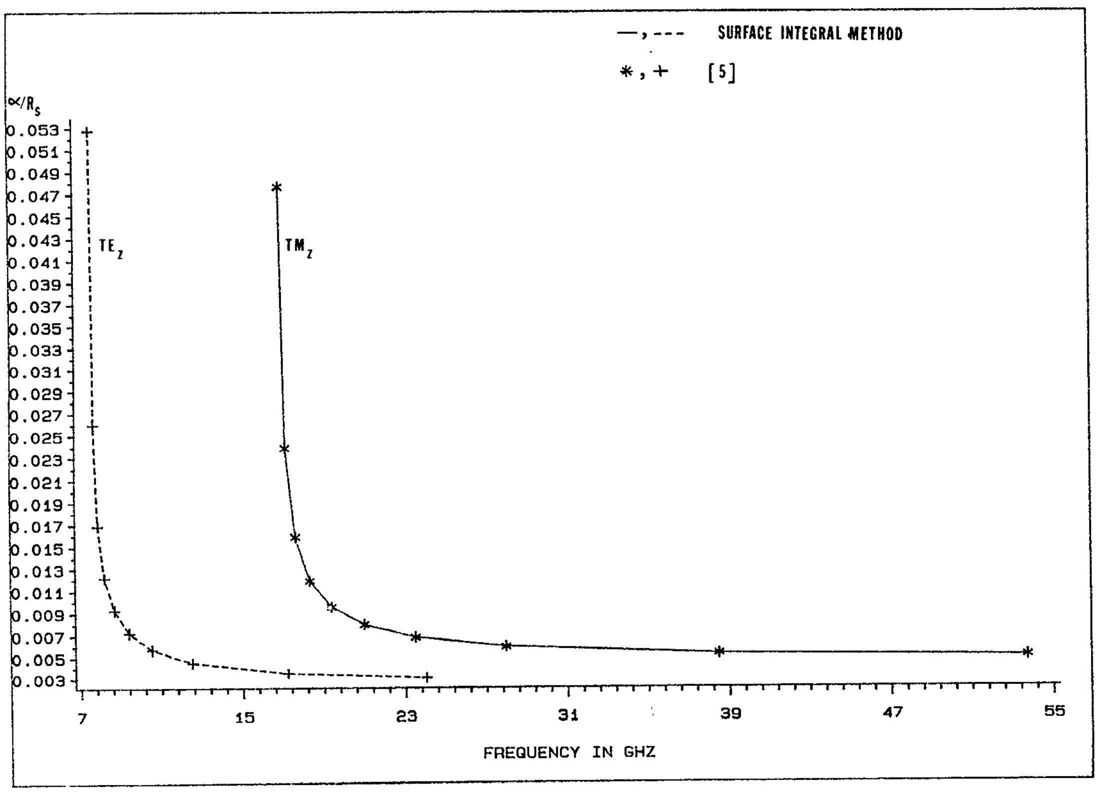

Fig. 2: Attenuation constant for the vectangular waveguide.

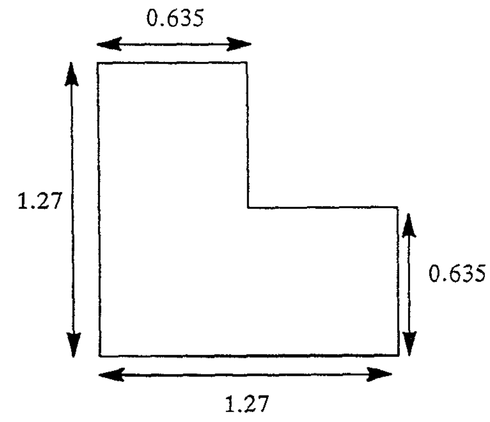

Fig. 3: L-shaped waveguide

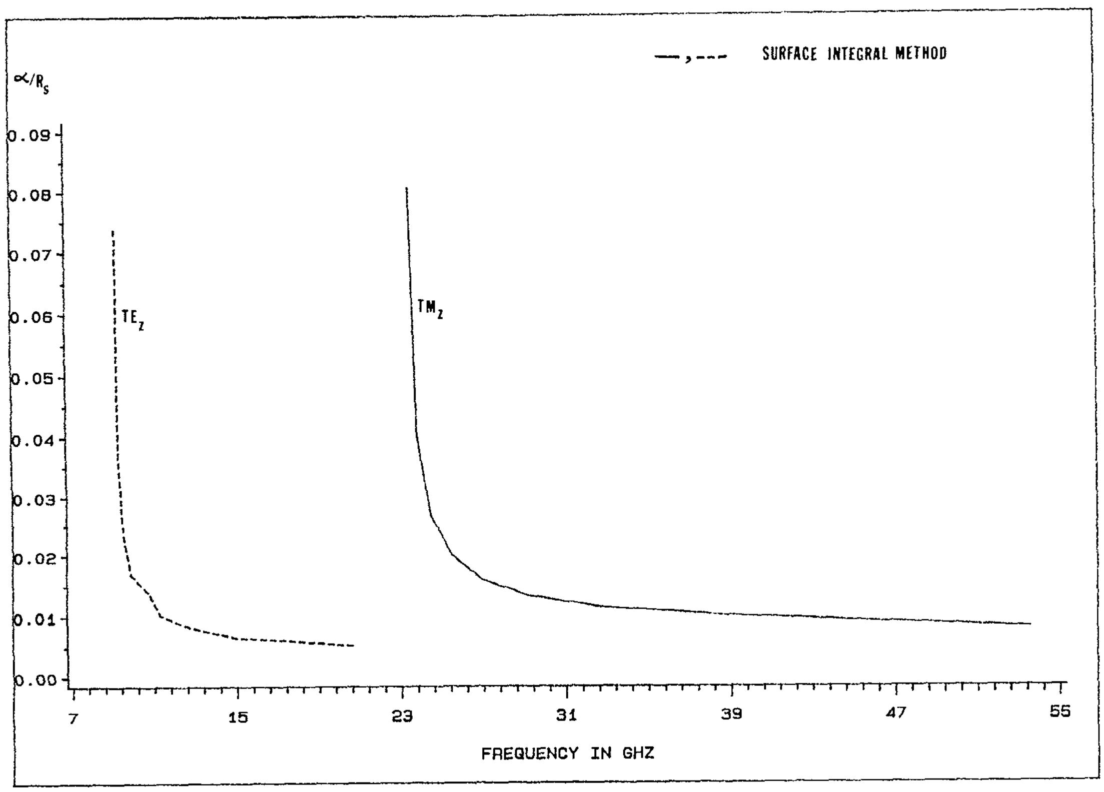

Fig. 4: Attenuation constant for the L-shaped waveguide